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MARK G. FILLER AND JAMES A. DIGABRIELE

SHORT-TERM SALES USING A SEASONAL

FORECASTING ADJUSTMENT MODEL

LOSSES SUSTAINED DUE TO A BUSINESS INTERRUPTION CAN BE DETERMINED BY USING A TIME-SERIES MODEL TO PREDICT WHAT SALES WOULD HAVE BEEN BUT FOR THE CASUALTY.

Often in the arena of business interruption claims and commercial damages, whether arising from breach of contract or tort, the valuation analyst has the need for a reliable short-term sales forecasting tool. Unlike sales forecasting for business valuation purposes where time units are measured in years, whether it is just next year's sales or a period of up to five-to-ten years until sales stabilize, commercial damages forecasts often are measured in months, or even weeks. To accommodate this need, a body of knowledge has been developed that presents itself in two formats—econometric, or explanatory, models and time-series models.

Explanatory Models vs. Time-Series Models

Explanatory models assume that the variable to be forecast exhibits an explanatory relationship with one or more independent variables. For example, gross domestic product (GDP) is a function of monetary and fiscal policies, inflation, capital spending, imports, exports, and error. This relationship is not exact. There will always be changes in GDP that cannot be accounted for by the variables in the model, and thus some part of GDP changes will remain unpredictable. Therefore, an "error" term is included, representing random effects beyond the variables in the model that affect the GDP figures.

Explanatory models can be applied to many systems, e.g., a national economy, a company's market, or a company's sales. The purpose of the explanatory model is to discover the form of the relationship and use it to forecast future values of the forecast variable. According to explanatory forecasting, any change in inputs will affect the output of the system in a predictable way, assuming the explanatory relationship has not changed.

Unlike explanatory forecasting, time-series forecasting treats the system as a black box and makes no attempt to discover the factors affecting its behavior. Therefore, a prediction of the future is based on past values of a variable, but not on explanatory variables that may affect the system. The objective of such time-series forecasting methods is to discover the pattern or trend in the historical data series and extrapolate that pattern or trend into the future.

There are three main reasons for wanting to treat a system as a black box:

1. The system may not be understood, and even if it were understood, it might be extremely difficult to measure the relationships assumed to govern its behavior.
2. The main concern may be only to predict what will happen and not know why it happens.
3. It might be reasonable to assume that changes in a time series are but proxies for all the explanatory variables that drive the system through repeated transaction patterns over time.

For example, if the only purpose were to forecast future values of GDP without concern as to why a certain level of GDP will be realized, a time-series approach would be appropriate. It is known that the magnitude of GDP does not change drastically from one month to another, or even from one year to the other. Thus the GDP of the next month will depend on the GDP of the previous month and possibly that of the months before. This makes the job of forecasting next month's GDP relatively easy because it requires no special input values as is required by explanatory forecasting for GDP. In fact, all that is needed is a reasonable amount of past GDP monthly history.

Business Interruption Example

This article will deal with the application of a time-series model to an actual business interruption situation, in which a hypothetical discount department store retailer (Rollie's) suffered a fire loss during the months of November 2006, December 2006, and January 2007. The assignment is to determine the actual loss sustained by the insured for those months, which in turn requires a prediction as to what sales for those months would have been, but for the casualty.

Time Series. The historical data consists of a sequence of observations from November 2001 though October 2006, a period of sixty months. This sequence is called a time series, and the data set for Rollie's is shown in Exhibit 1.

Some of the variation in a time series may be due to the variation in the number of days in each month. It is a good idea to adjust for this known source of variation in order to allow the study of other interesting features. Month length can have quite a large effect, because length can differ by as much as 10% ((31 - 28) / 30). If this is not removed, it can affect seasonal patterns. Leap years, which come every four years [((365 x 3 + 366) / 4) = 365.25], are easily adjusted for by multiplying each month's sales by: ((365.25 / 12) / number of days for that month). For exam-

MARK G. FILLER, CPA/ABV, CBA, AM, CVA, is the founder and president of Filler & Associates, P.A., a CPA firm in Portland, Maine. His personal practice focuses on business valuations, measurement of damages for lost profits, and assistance in adjusting business interruption losses. JAMES A. DiGABRIELE, D.P.S., CPA/ABV, CFE, CFSA, DABFA, Cr.FA, CVA, is an assistant accounting professor at the School of Business at Montclair State University in Montclair, New Jersey.

ple, November 2001 would be adjusted as follows: $164,330 x ((365.25 / 12) / 30) = $164,330 x 1.014583 = $166,726. The adjusted time-series data set for Rollie's is shown in Exhibit 2. Also shown in Exhibit 2 is the percentage change from the prior year, indicating an upward trend, but with a rate of change that is diminishing over time.

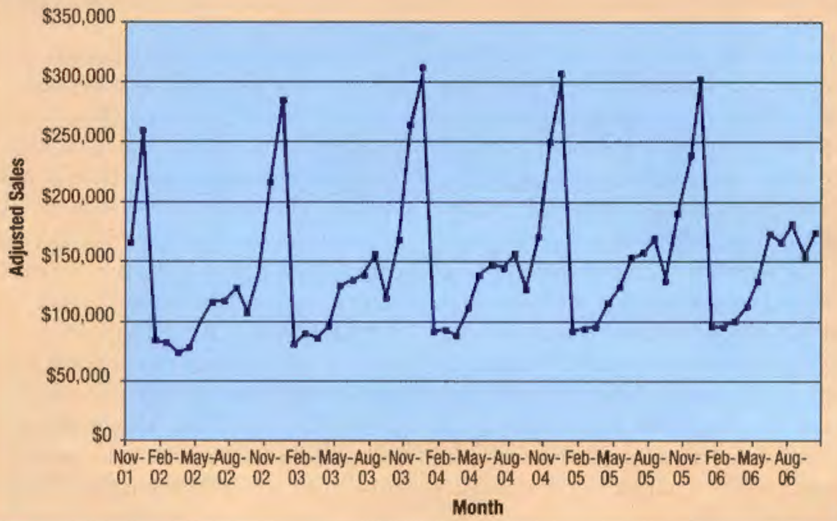
EXHIBIT 1
Rollie's Discount Department Store—
Actual Monthly Sales: November 2001-October 2006

	2001/2002	2002/2003	2003/2004	2004/2005	2005/2006
November	$164,330	$213,999	$260,667	$246,244	$235,708
December	264,400	291,119	311,513	305,520	308,677
January	84,906	82,849	93,205	93,430	96,997
February	75,390	82,485	88,736	86,226	88,157
March	75,328	87,838	90,085	97,369	101,207
April	77,102	94,337	109,497	113,309	111,176
May	101,519	132,548	141,435	131,180	136,851
June	114,315	132,788	145,444	150,194	170,377
July	119,700	141,325	147,984	160,488	168,906
August	130,284	158,056	158,522	171,851	184,999
September	105,470	119,961	124,592	134,631	152,787
October	143,399	172,229	174,212	194,255	178,025
Total	$1,456,143	$1,709,534	$1,845,892	$1,884,697	$1,933,867

EXHIBIT 2
Rollie's Discount Department Store—
Adjusted Monthly Sales: November 2001-October 2006

	2001/2002	2002/2003	2003/2004	2004/2005	2005/2006
November	$166,726	$217,120	$264,468	$249,835	$239,145
December	259,602	285,837	305,861	299,976	303,076
January	83,365	81,346	91,514	91,735	95,237
February	79,127	89,666	96,461	93,732	92,527
March	73,961	86,244	88,450	95,602	99,371
April	78,226	95,713	111,094	114,961	112,797
May	99,677	130,143	138,869	128,800	134,368
June	115,982	134,724	147,565	152,384	172,862
July	117,528	138,761	145,299	157,576	165,841
August	127,920	155,188	155,646	168,733	181,642
September	107,008	121,710	126,409	136,594	155,015
October	140,797	169,104	171,051	190,730	174,795
Total	$1,449,921	$1,705,555	$1,842,686	$1,880,659	$1,926,676
% Change from Prior Year		17.6%	8.0%	2.1%	2.4%

EXHIBIT 3
Time Plot



Visualizing Data Through Graphs. The single most important thing one can do when first exploring the data is to visualize the data through the use of graphs. The basic features of the data, including patterns, trends, and unusual observations, are most easily seen in graphic form. Graphs can also suggest possible explanations for some of the variation in the data.

The type of data will determine which type of graph is the most appropriate. For time series, the most obvious form is a time plot, in which data are plotted over time. For Rollie's, five years

EXHIBIT 4
Seasonal Plot

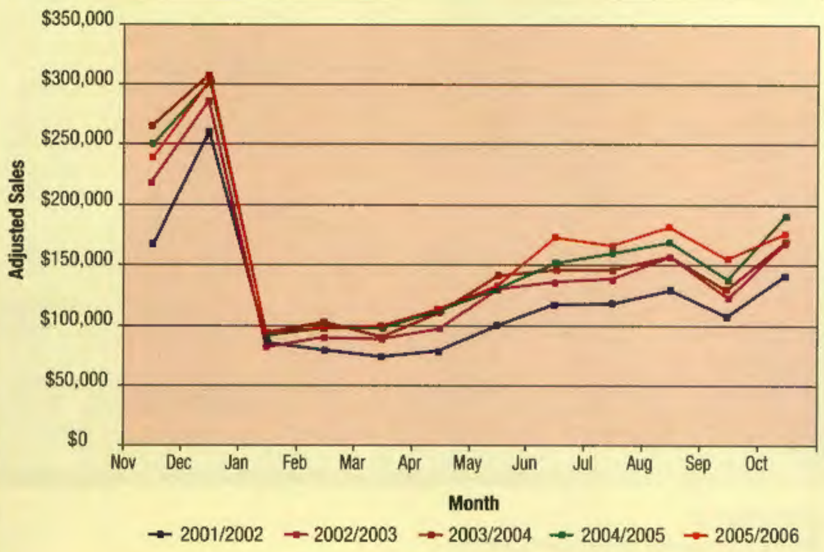
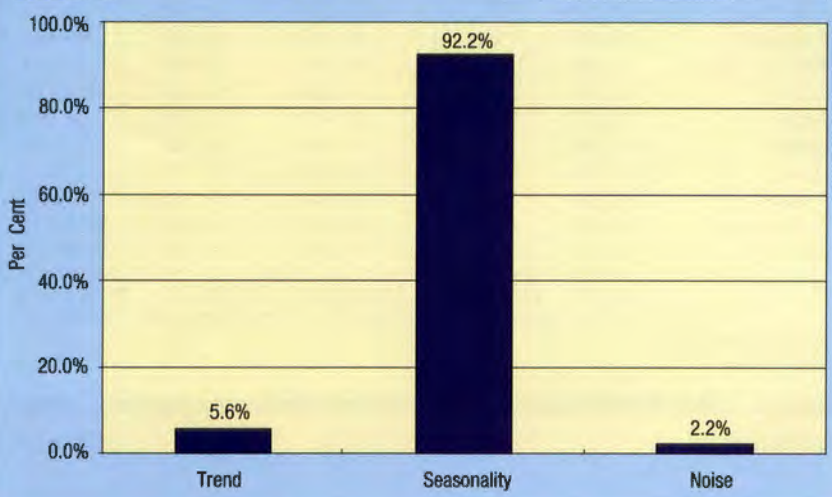


EXHIBIT 5
Analysis of Variance—Monthly data



of adjusted monthly sales data was graphed on Exhibit 3. This time plot immediately reveals seasonal behavior, which is a regular repeating pattern in the data. Along with seasonality, there is a systematic trend feature indicated by the fact that each year's succeeding data points are slightly higher than the year before. Trend is the long-term sweep or general direction of movement in a time series. It reflects the net influence of long-term factors that affect the time series in a fairly consistent and gradual way over time. In other words, the trend reflects changes in the data that occur with the passage of time.

For time-series data that are seasonal, it is often useful also to produce a seasonal plot. Exhibit 4 shows a seasonal plot of Rollie's adjusted sales. This graph consists of the data plotted against the individual "seasons" in which the data are observed. (In this case a "season" is a month.) This is something like a time plot, except that the data from each season are overlapped. A seasonal plot enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified. In this case, there are no departures from the overall pattern, and one can clearly see that each succeeding year's sales are higher than the year before.

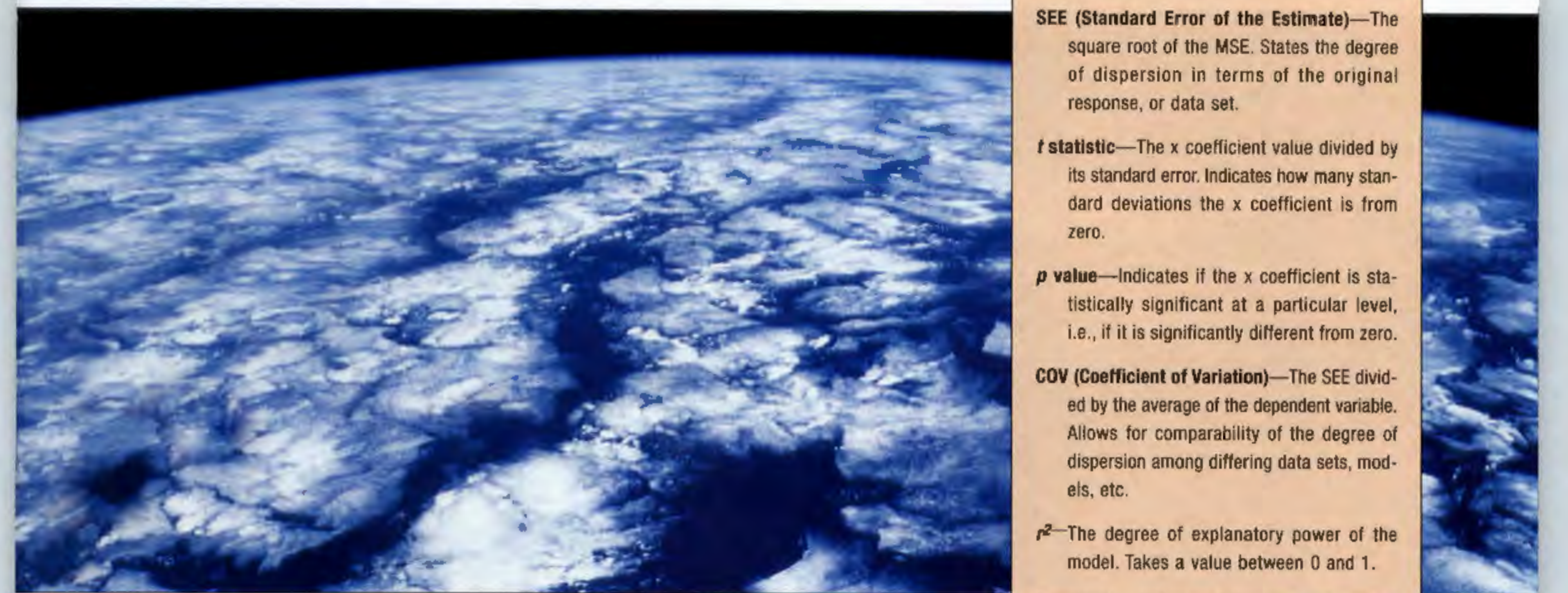
The visual conclusion that there was a great deal of seasonality and a slight trend pattern was confirmed by an analysis of variance, shown in Exhibit 5, which indicates the degree of trend, seasonality, and noise in a summary fashion. Here is a quick rundown of the calculations:

1. The grand mean, the mean of all the data points, was computed.
2. The squared difference between each data point and the grand mean was computed. This is defined as total variance.
3. The trend variance was computed, which is the sum of squared differences between the average value for each year and the grand mean. The ratio (12 x trend variance) / (total variance) is defined as the proportion of variance due to trend.
4. The seasonal variance was computed, which is the sum of squared differences between the average value for each month and the grand mean. The ratio (number of years of data x seasonal variance) / (total variance) is defined as the proportion of variance due to seasonality.
5. Because the proportion must add up to 1.0, the proportion of noise was taken to be 1.0 - (proportion due to trend + proportion due to seasonality).

The implication is that, for the business interruption claim at hand, a seasonal pattern, as well as a trend pattern, would need to be accounted for in the statistical model.

Regression Analysis. The next step, then, is to choose a statistical model that will account for both seasonality and a trend pattern, albeit a patern that is diminishing over time. Because it is a nonstationary time series, in which there is some upward trend in the data over time, forecasting techniques, such as the moving average, weighted moving average, and exponential smoothing will consistently underestimate the predicted values. This underestimate is due to the fact that the thechniques use some average of the previous values to forecast future values, Therefore, one needs to consider a technique appropriate for a nonstationary time series with an upward trend in the data over time, coupled with a seasonal pattern.

Such a technique is regression analysis. A regression model of a time series can be built if data are available for one or more independent variables that account for the systematic movements of the time series. However, even if no independent variables have a *causal* relationship with the time series, some independent variables might have a *predictive* relationship with the time series. A predictor variable does not have a cause-and-effect relationship with the time series. However, the behavior of a predictor variable might be correlated with that of the time series in a way that helps to forecast future values of the time series.



As mentioned earlier, trend is the long-term sweep or general direction of movement in a time series that reflects changes in the data over time. The mere passage of time does not cause the trend in the time series. However, like the consistent passage of time, the trend of a time series reflects the steady upward or downward movement in the general direction of the series. Thus, time itself might represent a predictor variable that could be useful in accounting for the trend in a time series.

If a linear trend line were superimposed on the time plot graph in Exhibit 3, one would see that while the line accounted for the upward trend in the data, the actual values would not appear to be scattered randomly around the trend line, a basic requirement for a good fitting regression-derived trend line. Any one of the observations is more likely to be substantially above the line or only slightly below the line. This suggests that a linear trend model might not be appropriate for this data.

As an alternative, a curved, or curvilinear, trend line will produce a better fit with the data, as an upward trend that is diminishing over time must be accounted for. A model that will serve this purpose is a quadratic, or second-degree polynomial, model, in which time is the first degree and time

GLOSSARY

MSE (Mean Squared Error)—The sum of the squared difference between the actual response and the predicted response divided by the error degrees of freedom. Measures the degree of dispersion in the data set.

Degrees of freedom—Number of data points less the number of regression parameters, including the constant.

SEE (Standard Error of the Estimate)—The square root of the MSE. States the degree of dispersion in terms of the original response, or data set.

***t* statistic**—The x coefficient value divided by its standard error. Indicates how many standard deviations the x coefficient is from zero.

***p* value**—Indicates if the x coefficient is statistically significant at a particular level, i.e., if it is significantly different from zero.

COV (Coefficient of Variation)—The SEE divided by the average of the dependent variable. Allows for comparability of the degree of dispersion among differing data sets, models, etc.

r^2—The degree of explanatory power of the model. Takes a value between 0 and 1.

F statistic—Indicates if the model as a whole is statistically significant.

Grand mean—The mean of several subgroups; in this case, the average of the 12 monthly averages.

Root Mean Square Error (RMSE)—A measure of forecast accuracy that has the same advantage of the MSE in that a penalty is assessed to large forecast errors, but the value of RMSE is comparable in magnitude to other commonly used statistics for forecast accuracy.

Mean Absolute Percentage Error (MAPE)—A measure of forecasting accuracy that is used whenever knowledge of the size of the forecast error in relation to the size of the variable to be forecast is important.

Mean Percentage Error (MPE)—A measure of forecast accuracy that is used whenever the forecaster is interested in determining whether the forecast error is biased.

Mean Absolute Deviation (MAD)—A measure of forecasting accuracy that is used whenever the forecaster wishes to obtain a measure of the forecast error that is expressed in magnitudes comparable to the original observations.

squared is the initial second degree of the regression equation. Later, it will be shown how the second degree can be modified to be any exponent that helps produce a better fitting model.

Seasonal Indices. Now that it has been determined how trend will be accounted for, an effective way of modeling seasonal effects in a time series must be found. A simple way to do this is by use of "dummy," or indicator, variables for each of the months in a year. The downside to this method is that it uses up ten more degrees of freedom (see glossary) than necessary; thus, another approach, known as seasonal indices, will be chosen. This approach, while more complex to develop, saves those precious degrees of freedom, which, all else being equal, allows for a smaller standard error of the estimate.

Seasonal indices reflect the average percentage by which observations in each "season" (month) differ from their projected trend values. For example, Rollie's sales in December are above the value predicted using a trend model, while January's sales fall below the

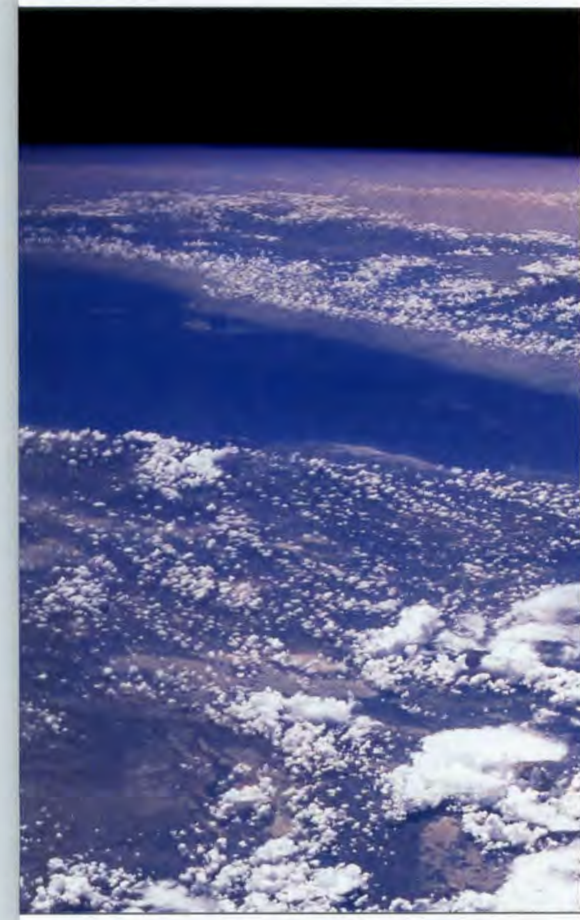


EXHIBIT 6
Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales

Year	Month	Time Period	Time Period^2	Adjusted Sales	Quadratic Trend
2001	11	1	1	$166,726	123,352
	12	2	4	259,602	124,549
2002	1	3	9	83,365	125,725
	2	4	16	81,953	126,881
	3	5	25	73,961	128,016
	4	6	36	78,226	129,130
	5	7	49	99,677	130,223
	6	8	64	115,982	131,295
	7	9	81	117,528	132,347
	8	10	100	127,920	133,378
	9	11	121	107,008	134,388
	10	12	144	140,797	135,377
	11	13	169	217,120	136,345
	12	14	196	285,837	137,293
2006	1	51	2601	95,237	157,729
	2	52	2704	95,831	157,886
	3	53	2809	99,371	158,023
	4	54	2916	112,797	158,138
	5	55	3025	134,368	158,233
	6	56	3136	172,862	158,307
	7	57	3249	165,841	158,360
	8	58	3364	181,642	158,392
	9	59	3481	155,015	158,404
	10	60	3600	174,795	158,394
				MSE	3,544,437,094

EXHIBIT 7
Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales With Percentage of Trend

Year	Month	Time Period	Time Period^2	Adjusted Sales	Quadratic Trend	Adjusted as a % of Trend
2001	11	1	1	$166,726	123,352	135.2%
	12	2	4	259,602	124,549	208.4%
2002	1	3	9	83,365	125,725	66.3%
	2	4	16	81,953	126,881	64.6%
	3	5	25	73,961	128,016	57.8%
	4	6	36	78,226	129,130	60.6%
	5	7	49	99,677	130,223	76.5%
	6	8	64	115,982	131,295	88.3%
	7	9	81	117,528	132,347	88.8%
	8	10	100	127,920	133,378	95.9%
	9	11	121	107,008	134,388	79.6%
	10	12	144	140,797	135,377	104.0%
	11	13	169	217,120	136,345	159.2%
	12	14	196	285,837	137,293	208.2%
2006	1	51	2601	95,237	157,729	60.4%
	2	52	2704	95,831	157,886	60.7%
	3	53	2809	99,371	158,023	62.9%
	4	54	2916	112,797	158,138	71.3%
	5	55	3025	134,368	158,233	84.9%
	6	56	3136	172,862	158,307	109.2%
	7	57	3249	165,841	158,360	104.7%
	8	58	3364	181,642	158,392	114.7%
	9	59	3481	155,015	158,404	97.9%
	10	60	3600	174,795	158,394	110.4%

EXHIBIT 8
Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales With Percentage of Trend and Seasonal Index

Year	Month	Time Period	Time Period^2	Adjusted Sales	Quadratic Trend	Adjusted as a % of Trend	Month	Seasonal Index
2001	11	1	1	$166,726	123,352	135.2%	1	61.4%
	12	2	4	259,602	124,549	208.4%	2	62.6%
2002	1	3	9	83,365	125,725	66.3%	3	60.7%
	2	4	16	81,953	126,881	64.6%	4	69.6%
	3	5	25	73,961	128,016	57.8%	5	85.7%
	4	6	36	78,226	129,130	60.6%	6	97.5%
	5	7	49	99,677	130,223	76.5%	7	97.4%
	6	8	64	115,982	131,295	88.3%	8	105.7%
	7	9	81	117,528	132,347	88.8%	9	86.3%
	8	10	100	127,920	133,378	95.9%	10	112.8%
	9	11	121	107,008	134,388	79.6%	11	158.0%
	10	12	144	140,797	135,377	104.0%	12	202.4%
	11	13	169	217,120	136,345	159.2%		
	12	14	196	285,837	137,293	208.2%		
2006	1	51	2601	95,237	157,729	60.4%		
	2	52	2704	95,831	157,886	60.7%		
	3	53	2809	99,371	158,023	62.9%		
	4	54	2916	112,797	158,138	71.3%		
	5	55	3025	134,368	158,233	84.9%		
	6	56	3136	172,862	158,307	109.2%		
	7	57	3249	165,841	158,360	104.7%		
	8	58	3364	181,642	158,392	114.7%		
	9	59	3481	155,015	158,404	97.9%		
	10	60	3600	174,795	158,394	110.4%		

value predicted using a trend model. Thus, if seasonal indices representing the average amount by which observations in a given month fall above or below the trend line can be determined, the trend projections could be multiplied by these amounts and the accuracy of the forecasts increased.

The next step, before the forecasting model can be set up, is to calculate seasonal indices for a quadratic trend mod-

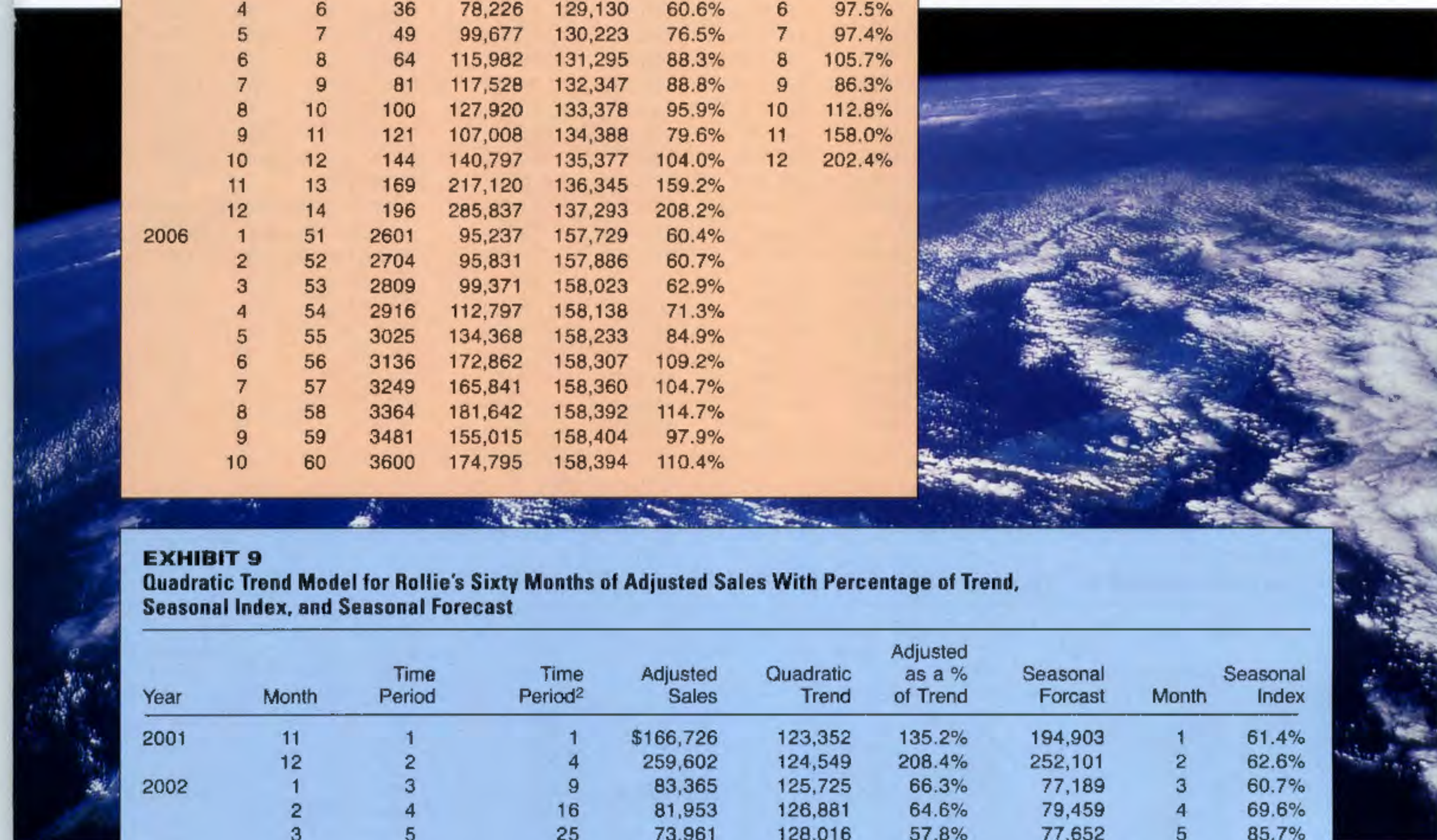


EXHIBIT 9
Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales With Percentage of Trend, Seasonal Index, and Seasonal Forecast

Year	Month	Time Period	Time Period^2	Adjusted Sales	Quadratic Trend	Adjusted as a % of Trend	Seasonal Forcast	Month	Seasonal Index
2001	11	1	1	$166,726	123,352	135.2%	194,903	1	61.4%
	12	2	4	259,602	124,549	208.4%	252,101	2	62.6%
2002	1	3	9	83,365	125,725	66.3%	77,189	3	60.7%
	2	4	16	81,953	126,881	64.6%	79,459	4	69.6%
	3	5	25	73,961	128,016	57.8%	77,652	5	85.7%
	4	6	36	78,226	129,130	60.6%	89,896	6	97.5%
	5	7	49	99,677	130,223	76.5%	111,587	7	97.4%
	6	8	64	115,982	131,295	88.3%	128,029	8	105.7%
	7	9	81	117,528	132,347	88.8%	128,938	9	86.3%
	8	10	100	127,920	133,378	95.9%	140,973	10	112.8%
	9	11	121	107,008	134,388	79.6%	115,979	11	158.0%
	10	12	144	140,797	135,377	104.0%	152,638	12	202.4%
	11	13	169	217,120	136,345	159.2%	215,433		
	12	14	196	285,837	137,293	208.2%	277,896		
2006	1	51	2601	95,237	157,729	60.4%	96,838		
	2	52	2704	95,831	157,886	60.7%	98,876		
	3	53	2809	99,371	158,023	62.9%	95,853		
	4	54	2916	112,797	158,138	71.3%	110,091		
	5	55	3025	134,368	158,233	84.9%	135,589		
	6	56	3136	172,862	158,307	109.2%	154,368		
	7	57	3249	165,841	158,360	104.7%	154,280		
	8	58	3364	181,642	158,392	114.7%	167,411		
	9	59	3481	155,015	158,404	97.9%	136,705		
	10	60	3600	174,795	158,394	110.4%	178,590		

MSE 89,415,966

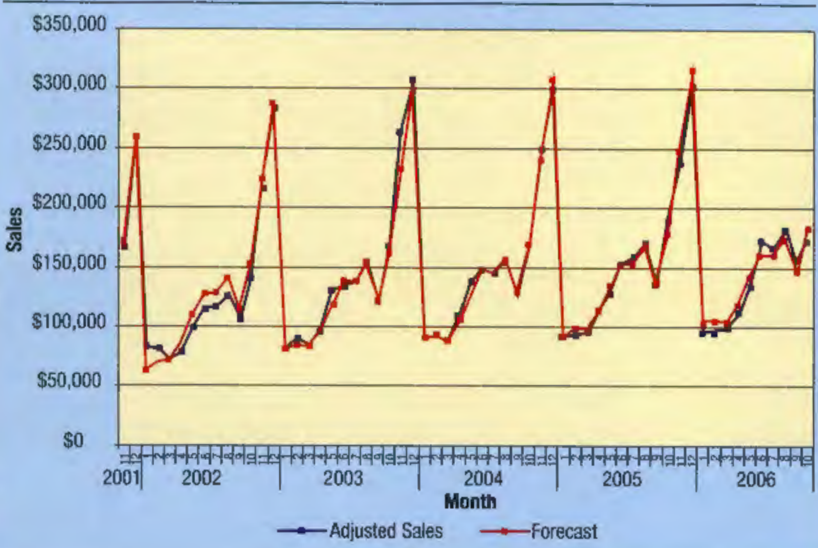
EXHIBIT 10
Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales With Refined Seasonal Adjustment Factors

Year	Month	Time Period	Time Period^2	Adjusted Sales	Quadratic Trend	Adjusted as a % of Trend	Seasonal Forcast	Month	Seasonal Index
2001	11	1	1	$166,726	117,973	141.3%	188,122	1	61.0%
	12	2	4	259,602	119,529	217.2%	240,252	2	61.9%
2002	1	3	9	83,365	121,056	68.9%	73,840	3	60.7%
	2	4	16	81,953	122,553	66.9%	75,821	4	70.1%
	3	5	25	73,961	124,021	59.6%	77,652	5	85.6%
	4	6	36	78,226	125,460	62.4%	87,902	6	97.9%
	5	7	49	99,677	126,870	78.6%	108,645	7	97.6%
	6	8	64	115,982	128,250	90.4%	125,534	8	105.7%
	7	9	81	117,528	129,601	90.7%	126,470	9	86.4%
	8	10	100	127,920	130,922	97.7%	138,446	10	112.5%
	9	11	121	107,008	132,215	80.9%	114,176	11	159.5%
	10	12	144	140,797	133,478	105.5%	150,103	12	201.2%
	11	13	169	217,120	134,711	161.2%	214,813	Average	100.0%
	12	14	196	285,837	135,915	210.3%	273,519		
2006	1	51	2601	95,237	159,849	59.6%	97,503	Intercept	116,386.98
	2	52	2704	95,831	159,939	59.6%	98,950	Slope 1	1,600.22
	3	53	2809	99,371	169,999	62.1%	97,134	Slope 2	(14.67)
	4	54	2916	112,797	160,030	70.5%	112,123		
	5	55	3025	134,368	160,032	84.0%	137,044		
	6	56	3136	172,862	160,004	108.0%	156,615		
	7	57	3249	165,841	159,947	103.7%	156,082		
	8	58	3364	181,642	159,860	113.6%	169,046		
	9	59	3481	155,015	159,744	97.0%	137,950		
	10	60	3600	174,795	159,599	119.5%	179,478		
						MSI	82,206,874		

EXHIBIT 11
Final Optimized Quadratic Trend Model for Rollie's Sixty Months of Adjusted Sales

Year	Month	Time Period	Time Period^2	Seasonal Index	Adjusted Sales	Exponent-iated Sales	Quadratic Trend	Month	Seasonal Index
2001	11	1	1.00000	159.5%	$166,726	42,693	170,281	1	61.0%
	12	2	0.46594	201.2%	259,602	63,223	260,547	2	61.9%
2002	1	3	0.29807	61.0%	83,365	23,091	64,935	3	60.7%
	2	4	0.21710	61.9%	81,953	22,744	70,146	4	70.1%
	3	5	0.16978	60.7%	73,961	20,766	71,207	5	85.6%
	4	6	0.13888	70.1%	78,226	21,824	85,769	6	97.9%
	5	7	0.11719	85.6%	99,677	27,056	108,883	7	97.6%
	6	8	0.10116	97.9%	115,982	30,946	127,546	8	105.7%
	7	9	0.08884	97.6%	117,528	31,311	128,279	9	86.4%
	8	10	0.07911	105.7%	127,920	33,754	141,056	10	112.5%
	9	11	0.07122	86.4%	107,008	28,813	114,313	11	159.5%
	10	12	0.06471	112.5%	140,797	36,750	152,665	12	201.2%
	11	13	0.05925	159.5%	217,120	53,958	223,664	Average	100.0%
	12	14	0.05460	201.2%	285,837	68,857	289,106		
2006	1	51	0.01314	61.0%	95,237	25,984	104,380	Sales Exp	0.88670
	2	52	0.01286	61.9%	95,831	26,128	106,182	Time^2 Exp	(1.10178)
	3	53	0.01260	60.7%	99,371	26,982	105,147		
	4	54	0.01234	70.1%	112,797	30,191	118,904		
	5	55	0.01209	85.6%	134,368	35,259	141,798		
	6	56	0.01185	97.9%	172,862	44,083	160,237		
	7	57	0.01163	97.6%	165,841	42,492	160,412		
	8	58	0.01140	105.7%	181,642	46,063	173,045		
	9	59	0.01119	86.4%	155,015	40,023	145,258		
	10	60	0.01099	112.5%	174,795	44,520	184,238		
						MSE	71,533,921		

EXHIBIT 12
Graph Showing Adjusted Sales vs. Forecasted Sales



el. Exhibit 6 shows the initial setup of the quadratic trend model for Rollie's sixty months of adjusted sales. (Years 1997-1999, rows 17-52 have been hidden for presentation purposes.) The formula for cell F3 is: =TREND(($E$3:$E$62, $C$3:$D$62,C3:D3) which is then copied down to and through row 62.

The accuracy of this forecast can be evaluated by seeing how well it explains past behavior of the time-series variables. One method, which will be introduced later, is to construct line plots that show the actual data versus the values predicted by the model. More formal quantitative measures of the accuracy of time-series modeling techniques are mean absolute deviation (MAD), the mean absolute percentage error (MAPE), the mean square error (MSE) and the root mean square error (RMSE). The MSE measure was chosen because it is somewhat easier to calculate than the other metrics. The formula for cell F64 is: =SUMXMY2(F3:F62,E3:E62)/COUNT(F3:F62) and it computes an MSE of 3,549,850,132, which is an exceedingly large number, as the calculated trend line traverses only the midpoint of the data set and does not account for seasonal fluctuations. In order to account for that, seasonal indices in the model need to be included.

The goal in developing seasonal indices is to determine the average percentage by which observations in each "season" (month) differ from the value predicted for them using the particular trend model selected. To accomplish this, in column G of Exhibit 7 the ratio of each actual value in column E to its corresponding projected trend value shown in column F was calculated as: =E3/F3, and then this formula was copied down to and through row 62. The results are shown on Exhibit 7.

The value in cell G3 indicates that the actual value in period 1 was 135.6% of (or approximately 35.6% larger than) its estimated trend value. The value in cell G5 indicates that the actual value in period 3 was 66.5% of (or approximately 33% smaller than) its estimated trend value. The remaining values in column G have similar interpretations.

The seasonal index for each month is obtained by computing the average of the values in column G on a month-by-month basis. For example, the seasonal index for month 11 equals the average of the cells in G3, G15, G27, G39, and G51. The seasonal index for month 12 equals the average of the values in cells G4, G16, G28, G40, and G52. Similar computations are required to calculate seasonal indices for months 1-10. Separate Excel AVERAGE () functions for each month can be used to compute these averages. However, for large data sets, such an approach would be tedious and prone to error. Thus, the averages shown in cells K3 through K14 are calculated as: =SUMIF(B3$B$62,J3,$G$3:$G$62)/COUNTIF($B$3:$B$62,J3), which is then copied down and through row 62. The results are shown in Exhibit 8.

The seasonal index for month 1 shown in cell K3 in Exhibit 8 indicates that, on average, the actual sales value in January of any given year will be 61.4% of (or 38.6% smaller than) the estimated trend value for the same period. Similarly, the seasonal index for month 8 shown in cell K10 indicates that, on average, the actual sales value in August will be 105.7% of (or 5.7% larger than) the estimated trend value for the same period. The seasonal indices for all the other months have similar interpretations.

The calculated seasonal indices can be used to refine or adjust the trend estimates. This is accomplished in column H of Exhibit 9 as: =F3*VLOOKUP (B3,$J$3:$K$14,2), which is then copied down through row 62. This formula takes the estimated trend value for each period and multiplies it by the appropriate seasonal index for the month in which the period occurs. The trend estimates for month 1 observations are multiplied by 61.4%, the trend estimates for month 2 are multiplied by 62.2%, and so on for months 3 through 12. Before a

EXHIBIT 13
Summary Output from Regression Tool

	Regression Statistics	
	Transformed	Back-Transformed
Multiple R	0.9898	0.9901
R Square	0.9797	0.9803
Adjusted R Square	0.9787	0.9792
Standard Error	2,017	8,793
Observations	60	60

ANOVA

	df	SS	MS	F	Significance F
Regression	3	11,021,252,254	3,673,750,751	903	2.31479E-47
Residual	56	227,929,305	4,070,166		
Total	59	11,249,181,559			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	925.379	893.368	1.036	0.305	(864.250)	2,715.009
Time Period	136.431	17.582	7.760	0.000	101.209	171.652
Time Period$^{-1.10178}$	(11,037.043)	2,182.390	(5.057)	0.000	(15,408.893)	(6,665.194)
SeasonalFactor	33,470.974	659.070	50.785	0.000	32,150.700	34,791.248

PREDICTION

Regression Equation: Forecasted Monthly Sales = (925.379 + Time Period x 136.431 + Time Period$^{[-1.10178]}$ x -11037.043 + Seasonal Factor x 33470.974)$^{[1/.886701]}$

Month	Intercept	Time Period	Time Period$^{[-1.10178]}$	Seasonal Factor	Transformed Forecast	Back-Transformed Forecast$^{[1/.886701]}$	Reconverted
Nov-06	925.379	61	0.01079	159.46%	62,502	256,268	252,585
Dec-06	136.431	62	0.01060	201.24%	76,625	322,458	328,417
Jan-07	-11037.043	63	0.01041	61.00%	29,822	111,243	113,298
Total							694,300

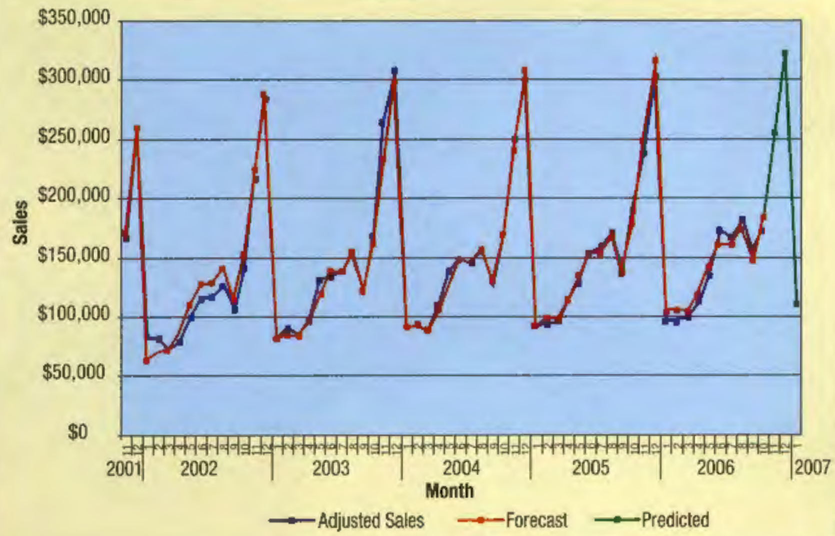
graph is created showing the actual sales data versus the seasonal forecast calculated in column H, the seasonal indices need to be refined. Cell H64 calculates MSE to be 89,444,067, an increase of 97.5% in accuracy from the pure quadratic trend model shown in Exhibit 6.

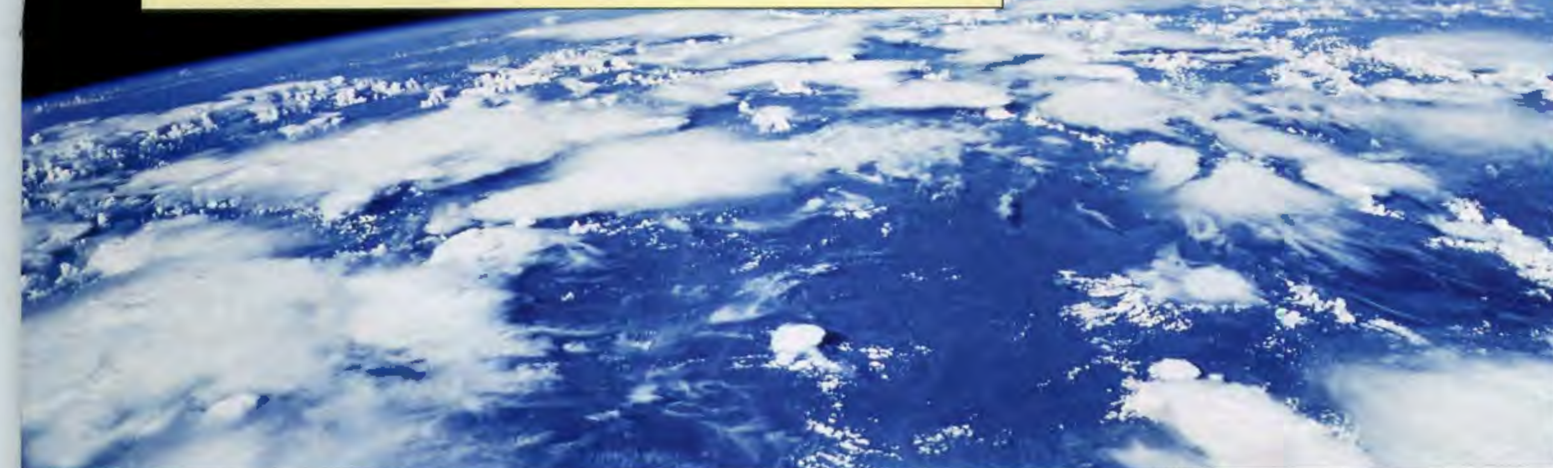
Refining the Model. While the approach for calculating seasonal indices illustrated in Exhibit 9 has considerable intuitive appeal, it is important to note that these seasonal adjustment factors are not necessarily optimal. The seasonal adjustment factors can be refined with a similar approach that uses Excel's Solver add-in to determine the optimal values of the seasonal indices and the parameters of the quadratic trend model simultaneously. In Exhibit 9, cells K17, K18, and K19 on the worksheet (K53, K54, and K55 on the truncated demonstration sheet) are used to represent, respectively, the estimated values of the intercept, the time coefficient, and the time^2 coefficient in the quadratic trend model. The cells in Exhibit 10 should be labeled as indicated, and the amounts 1,000, 500, and 100 in each cell should be entered as placeholders. The formula in cell F3 should be changed to: $K$17+$K$18*C3+$K$19*D3 and copied down to and through row 62. Next, Solver will be set up to find the optimal values for the trend and seasonal parameters that minimizes the MSE.

Under Tools, Solver, cell H64 should be selected and minimized by changing cells K3:K14 and cells K17:K19 while constraining cell K15 to a value of 1 (or 100%). This constraint is necessary because if the seasonal indices do not average to 100%, there is some upward or downward bias in the trend component of the model. Click Solve, and the results are shown on Exhibit 10. Notice that the MSE has been reduced to 82,530,189, an improvement in model accuracy of 7.7%.

While this is an improvement over the previous model, the model can be optimized even further by transforming by exponentiation the dependent variable, sales, and modifying the exponent of the second degree of the quadratic model, time^2. This is done because time-series data are: (1) rarely linear, (2) infrequently homogeneous as to variance, and (3) not often distributed normally, or even, symmetrically. Fortunately, these three problems can be fixed with one procedure, transformation of either or both the independent and dependent variables. Data that is not normally distributed is also often neither linear nor homogeneous. Thus, transformation pro-

EXHIBIT 14
Graph Showing Three-Month Prediction in Relation to 60-Month History





ly with the actual sales. In fact, the average distance between the actual and forecasted data points is only $8,793, which translates into a coefficient of variation of only 6%. This means that if sales for period 61 were predicted using this model, one could be 95% confident that the actual level of sales observed would fall somewhere in the range from $238,682 to $273,854 ($256,268 ± $17,586). More information concerning the model's metrics can be found on Exhibit 13.

Exhibit 13 presents Excel's summary output for its regression tool, and shows the computations for the expected sales during the three-month period of interruption, including the reconversion of sales back to actual days in a month. The regression statistics are all excellent, and the t-stats of the x coefficients are all way above 2, all of which indicate, along with the low standard error of the estimate, an extremely well fitting model with highly accurate results. The end result can be seen on Exhibit 14, which shows the three-month prediction in relation to the 60-month history on which it is based.

vides a simple way both to fix statistical problems (non-symmetrical, non-normal, and heterogeneous distributions) and to obtain a better fit of curves to data (curvilinear regression). To accomplish this Exhibit 10 must be modified.

Exhibit 11 is set up through the following steps:

1. Copy columns A, B, and C from Exhibit 10.
2. Enter the labels Sales Exp and Time^2 Exp and the placeholder amounts of 2 and 1 in cells J17:K18 (J53:K54 on the truncated demonstration sheet).
3. In cell D3, enter the formula: =C3^$K$18 and copy it down to and through row 62.
4. In column E, enter the appropriate seasonal index from cells K3:K14.
5. In column F bring over adjusted sales from column E on Exhibit 10.
6. In cell G3, enter the formula: =F3*$K$17 and copy it down and through row 62.
7. In cell H3, enter the formula: =TREND($G$3:$G$62,$C$3:$E$62,C3:E3)^(1/$K$17) and copy it down to and through row 62.
8. Enter the MSE formula in cell H64, with cells H3:H62 and F3:F62 as the necessary elements.
9. Under Tools, Solver, select cell H64 and minimize it by changing cells K17:K18, and then click Solve.

The results should look like Exhibit 11. The MSE has been reduced to 72,169,758, a 12.6% improvement over the model in Exhibit 10. The results of this model can now be used to produce a line plot graph in Exhibit 12 showing adjusted sales versus forecasted sales.

From Exhibit 12's line plot, one can see that the forecasted values match up close-

Conclusion

This article has presented a succinct method of predicting future values of a time-series variable when seasonal and trend patterns are present. The goal was to demonstrate how a model can be fitted to the past behavior of a time series and then be used to predict future values. ■