

## Econometric Forecasting in a Lost Profits Case

I read with interest the recent article by A. Frank Adams, III, Ph.D., in the May/June 2008 issue of *The Value Examiner*<sup>1</sup>. I applaud Dr. Adams for his attempt to educate your readers in the area of econometric, or explanatory forecasting, particularly by way of use of regression analysis, a tool all valuation analysts and forensic accountants need to become more familiar with. However, I believe that some fundamental analytics weren't applied to the data set, resulting in an estimate of lost revenue that a reasonable person might conclude is overstated. This might have resulted from the complex nature of his calculations that were necessary to demonstrate the use of dummy variables and explanatory, rather than plain revenue data, as well as the focus on lost sales rather than total sales during the period of interruption (POI).

While the purpose of his article was to impart knowledge, this knowledge must be more than academic – it must ultimately be practical, i.e., able to be used in the daily work of your readers. Therefore, I would like to suggest some improvements and refinements to Adams' model and then I will propose a different regression model that will give a practical solution to the problem, albeit forcing us to drop the explanatory model espoused by Adams and substituting an optimized seasonal adjustment time-series model.

The first thing one ought to do when dealing with any data set, especially time series data, of which the motel data is typical, is to graph that data. Creating a line chart with a trend line of the competition's occupancy percentage for the 25-month period March 2003 – March 2005 as shown on Exhibit A would indicate a great deal of seasonality as well as a downward trend of almost 1% per quarter. In fact, seasonality accounts for 95% of the variation in the data, with noise (unexplained randomness) accounting for just 5%, as shown on Exhibit B. Converting the subject motel's data into monthly sales, and then charting the 16-month period March 2003 – June 2004 gives similar results, as shown on Exhibits C and D. These exhibits demonstrate the strength of the seasonality factor, thereby requiring the analyst to come up with a way to directly incorporate it into a model.

A second thing that an analyst ought to do is to examine the regression output regarding the strength of the model to explain the variation in the dependent variable by way of variation in the independent variables. While the t-statistics for each of the independent variables in Adams' model indicates statistical significance at the 5% level, the adjusted  $R^2$  of the model at .6632 is marginal at best for financial data, especially time series data. Correspondingly, the standard error of the estimate is 8.38%, which produces a coefficient of variation of 11.1%, meaning that on average, any forecast of motel occupancy will be off by that amount. In addition, it indicates a high level of inaccuracy, as for example in Adams' calculation 1 in Table 4. At the 95% confidence

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<sup>1</sup> When a 'Simple' Analysis Won't Do: Applying Economic Principles in a Lost Profits Case

level the range of error is approximately<sup>2</sup>  $\pm 16.76\%$ , producing a bell-shaped distribution whose motel occupancy ranges from 62.6% to 96.1%.

Per his Table 5, Adams' regression model predicts total lost revenue of \$223,329 during the 7-month POI. Since actual revenues for that period were \$797,111, this makes his predicted revenue for the 7-month POI \$1,020,440, an increase of \$68,008 over the corresponding period of the prior year. However, when we graph motel and competitors' occupancy percentage for the 16-month period leading up to the POI, as shown on Exhibit E, and then create a trendline for both sets of data, we find that the occupancy percentage is trending downward at the rate of .90% and .28% per month, respectively, a trend that continues on for the competitors through the whole 25-month period as seen on Exhibit A. Applying the same procedure to room revenue, as shown on Exhibit F, the seasonally unadjusted downward trends are \$1,193 and \$837 per month, respectively. How can we rely on a model that produces a 7.1% increase in revenue over the prior year when all indications are that, *ceteris paribus*, revenue should be less, not more? Room rates will not help solve the dilemma, as their trend lines are almost flat, increasing only at the rate of \$.16 and decreasing at the modest rate of \$.03 per month, respectively, as shown on Exhibit G.

Another questionable fact that isn't dealt with is the increase in competitors' occupancy percentage during the POI, which increase is used to predict the subject motel's occupancy percentage. It stands to reason that the competitors' occupancy increased by some unknown amount because the subject motel lost the use of 14 rooms each night for 7 months, and that its customers, therefore, ended up staying with its competitors. By ignoring this fact, a virtuous circular reference has been created, almost guaranteeing the overstatement of lost revenue.

The one indicator that would tip off an analyst to the problems with the original model presented in the article is the low adjusted  $R^2$  of .6632. This means that 66.32% of the variation in motel occupancy percentage is explained by the variation in all the independent variables, leaving 33.68% unexplained. Creating a correlation matrix would not have been of help in discerning the cause of the problem, as all of the correlations, while very low for business data, are still statistically significant at the 5% level. High t-statistics and low correlations indicate another viewpoint is needed. If each of the independent variables (x) in the model is to be a good predictor, it must have good accuracy, which means that the range of y must be small for each x variable. A scatterplot matrix, like the one in Exhibit H, would have shown that a more accurate prediction was impossible to obtain, as none of the relationships is strong enough to produce a narrow range of y-values for each x. All indications point to the conclusion that the model is missing one or more explanatory variables.

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<sup>2</sup> I say approximately because there is a formula, found in any statistics text, which will calculate precise prediction intervals based on the standard error for each prediction. This more correct method produces a slightly wider interval than that given above.

<sup>3</sup> Adjusted  $R^2$  compensates for independent variables that are included in the model but don't improve its goodness of fit. A well specified model will have an adjusted  $R^2$  only slightly less than  $R^2$ .

A statistically significant dummy variable indicates only that business was different from normal during the POI. While the use of a dummy variable to control, or account for, the decline in occupancy percentage during the POI is necessary for this model, it was not sufficient to capture the effects of a downward trend in room revenue and all the seasonality inherent in the data.

While Adams' model does include an independent variable that serves as a proxy for seasonality – competitors' occupancy percentage, as we have seen on Exhibit A, conveys the seasonality factor, albeit indirectly – it does not account for the downward trend in occupancy rates, and ultimately, room revenues. By adding a time trend variable and substituting a monthly seasonal index for competitors' occupancy rate, we can increase adjusted  $R^2$  by 13.9% and lower the standard error of the estimate by 14.8%. The value of the dummy variable coefficient drops from -.1814 to -.1352, causing expected revenue during the POI to fall from \$1,020,044 to \$963,759. See Exhibit I for the set-up and summary output of this model.<sup>4</sup> While these improvements are significant, they are not substantial, i.e., an adjusted  $R^2$  of .755 is still too low for time series data, and \$963,759 is still greater than the revenue in the same period of the prior year. We are left not knowing by how much competitors' occupancy rates are overstated during the POI, and at a loss as to what the missing explanatory variable(s) is.

An alternate way to calculate lost revenue is to simply forecast what revenue was expected to be during the POI and then to subtract actual revenue earned during the period. The proper model to use in this circumstance is one that will account for both the downward trends that were occurring as well as the seasonality of the hospitality industry. Just recently, I co-authored an article in *Valuation Strategies* that lays out, in a systematic manner, a technique that explicitly deals with these two issues<sup>5</sup>. Applying a slight variant<sup>6</sup> of the techniques suggested in that article to the current problem in the Adams article, we find that our optimized seasonally adjusted time-series model produces \$117,716 of lost revenue for the 7-month POI. Summary output metrics include an  $R^2$  of .904, a standard error of the estimate of \$6,490 (I computed predicted sales directly, rather than expected occupancy percentage), and a coefficient of variation of 4.71% (Adams' original COV of 11.1% is 253.3% larger). Forecasted revenue for the period is \$914,827, and when compared to revenue in the same time period in the prior year of \$952,432, shows a decrease of 3.9% that is reasonable given the downward trend in room revenue<sup>7</sup>. Both of these calculations are shown on Exhibit J.

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<sup>4</sup> While two of the independent variables have t-stats less than 2, in multiple regression, variables whose t-stats are greater than 1 are left in the model because to remove them would raise the standard error of the estimate and decrease the accuracy of the model's predictions.

<sup>5</sup> Filler and DiGabriele, "Short-Term Sales Forecasting Using a Seasonal Adjustment Model", *Valuation Strategies*, Vol. 11, No. 5, May/June 2008.

<sup>6</sup> The article sets forth a quadratic model to determine the monthly seasonal indices and exponentiated time trend and dependent variables in the forecasting model. Neither technique was appropriate for the motel case.

<sup>7</sup> For example, the subject motel's occupancy percentages for April, May and June of 2003 dropped from 95.6%, 95.8% and 96.2% to 77.4%, 82.8% and 84.9%, respectively for April, May and June of 2004. The constant monthly decrease for each of the 3 months is 1.75%, 1.20% and 1.04%, respectively, indicating a log-log relationship with time – the rate of decrease is decreasing at an ever-decreasing rate. Continuing

Exhibit K shows actual sales from March 2003 – June 2004, and then again for February and March of 2005. Forecasted sales are shown for the period March 2003 – June 2004, and then my projected sales from July 2004 – March 2005. Exhibit L repeats the information shown on Exhibit J, with the addition of a line indicating the sales projected for the period July 2004 – January 2005 as produced by Adams' original model. Exhibit M repeats the information shown on Exhibit J, with the addition of a line indicating the sales projected for the period July 2004 – January 2005 as produced by Adams' model as revised for time trend and direct seasonality. A close reading of these three exhibits indicates that my model captures the essence of the historical data and produces a forecast and projection that appears to reasonably follow past trends, while the original Adams model produces monthly room revenues that are far greater than one would expect given past history, and his revised model also produces forecasted revenues for the POI that are greater than the same period in the prior year, albeit in a lesser amount.

A reasonable question to ask at this point is: given the small number of data points available to us, how can we know if the revenue forecasts for the seven months of the POI produced by the two models are statistically different from each other? Although we know they are practically significant as the dollar difference is \$48,752 (\$963,579 - \$914,827), we would like to have some confidence that this difference is not the result of mere chance, i.e., the forecast couldn't go either way and therefore should not be subject to some averaging technique. Since both sets of forecasted sales depend on the particular seven months in question, and not just any seven months, they can be classified as dependent, or matched samples, and be made subject to the t-test for paired means. Our interest in the matched sample design is that since both forecasts are developed under similar conditions, (i.e., the same seven months), this design often leads to a smaller sampling error than the independent sample design. The primary reason for this is that the conditions prevalent in each month generate revenue data first under one forecasting methodology and then under the other methodology. Thus variation between months is eliminated as a source of sampling error. The setup data and output information for this t-test is shown on Exhibit N, which indicates that with a test statistic of -3.066 there is sufficient evidence to reject the null hypothesis at an  $\alpha$  level of .05.

One interesting element in the output is the Pearson correlation coefficient of .978, which indicates a strong linear relationship between the seven pairs of data values. This is what we would have expected given that each matched pair is driven by the circumstances prevailing in each of the months. Therefore, we can conclude that we have matched our data pairs on a relevant extraneous factor, i.e., the particular month of the POI, and have therefore applied the correct t-test. The results of this test, that both models cannot come from the same population, reinforce our logical argument that forecasted sales during the POI ought to be less than same period of the prior year, an argument that our seasonal regression model confirms.

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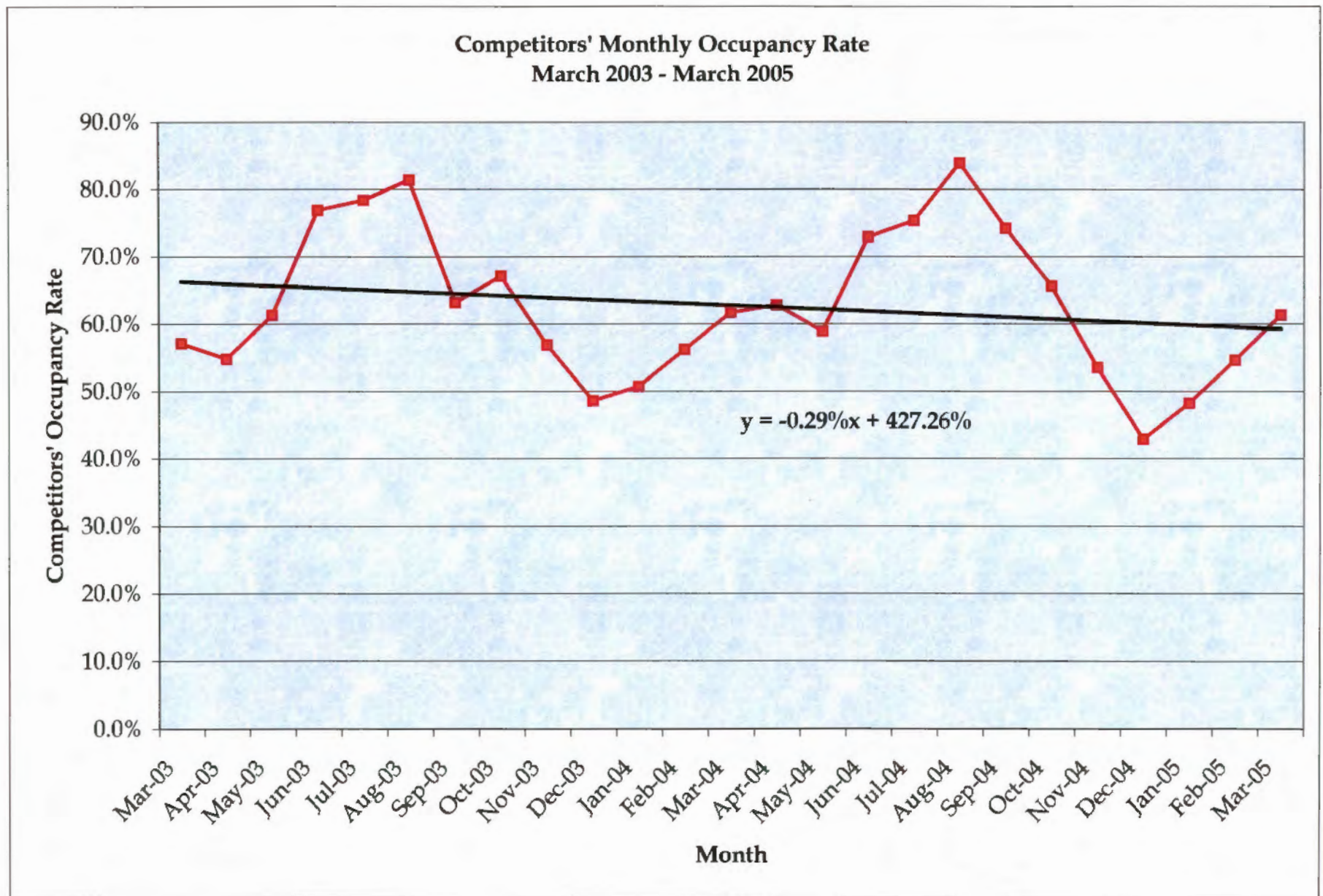
this trend over the next 7 months (July 2004 – January 2005) produces an expected total decrease of 4.9% during the POI. Cf. the forecasted decrease of 3.9%.

What we have just reviewed is a situation that often exists in a litigation setting – a trier of fact who wants and needs a number and who couldn't care less whether or not we have "sufficient" data with which to supply that answer. We are asked to form an opinion, "within a high degree of (economic) certainty", rather than establish truth for the expansion of knowledge. Each financial expert must do the best they can with what they have to work with, and convince the trier of fact that their number is the more reasonable answer to the question of damages. We cannot claim precision as our conclusions are subject to challenge – however, we must be as accurate as possible in the use of appropriate methods of analysis. I believe that the result I have supplied via a more direct approach, using revenue as the dependent variable and concurrently accounting for seasonality and trend gives an answer that comports well with past trends and that yields first-rate goodness-of fit metrics.

Mark G. Filler, CPA/ABV, CVA, CBA, AM  
March 2, 2009

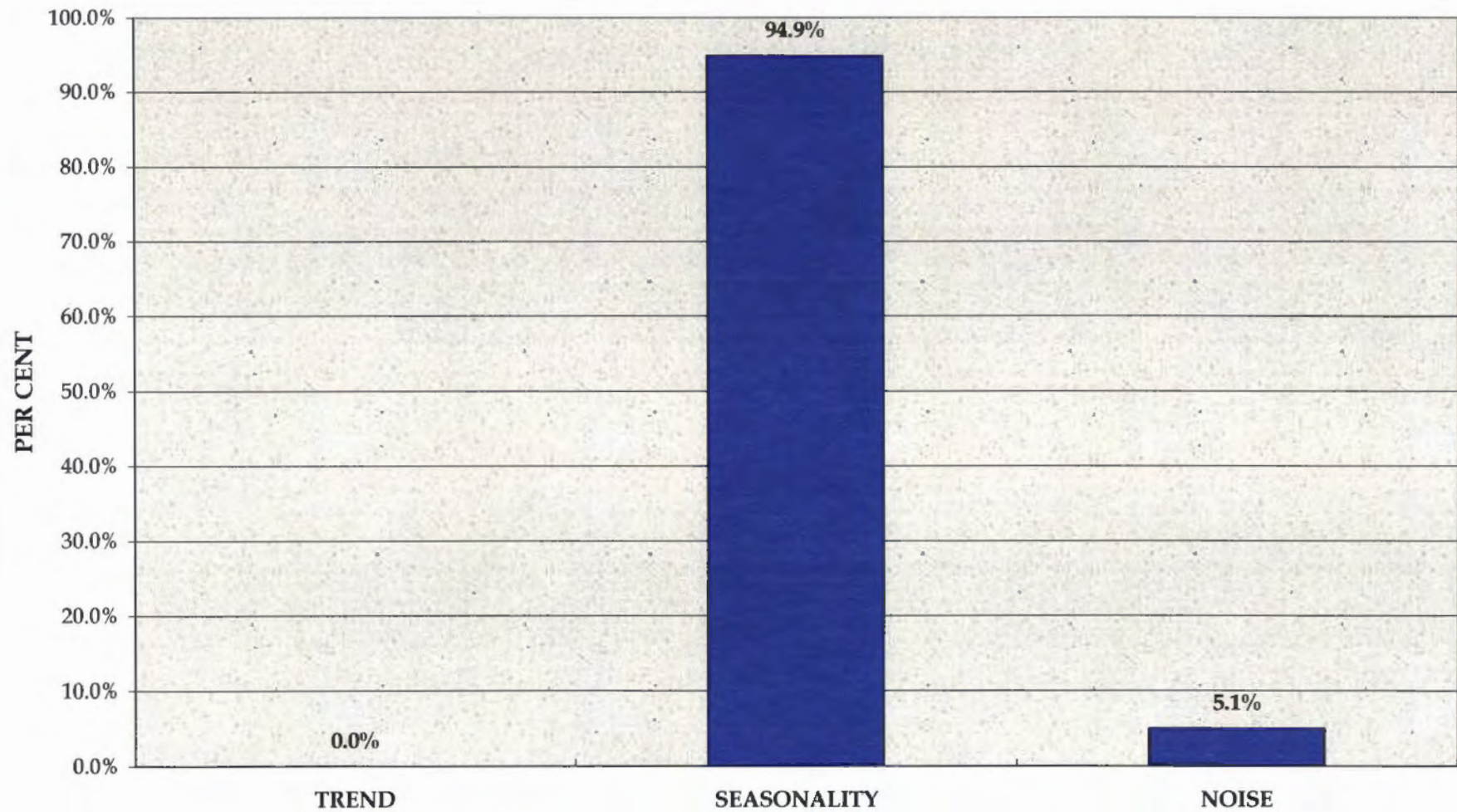


Exhibit A



## Exhibit B

### Competitors' Occupancy Rate March 2003 - February 2005



Analysis of Variance - Monthly Data

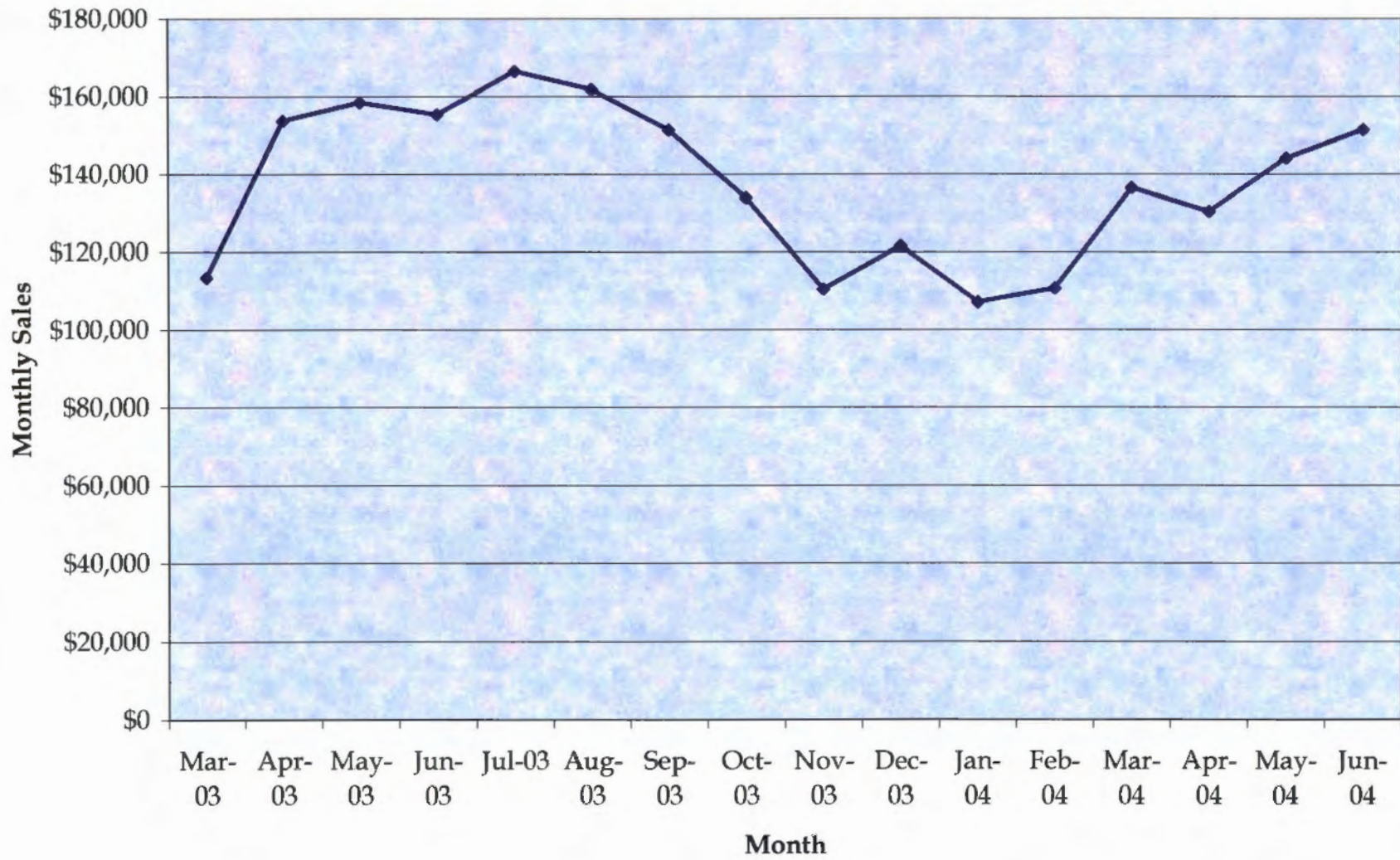
**Exhibit C**

Schedule of Monthly Room Revenue								
	A	B	C	D	E	F	G	H
Month	No. of Rooms	Days in Month	Motel Occupancy	Motel Room Rate	A * B * C * D Motel Sales	Competitors' Room Rate	Competitors' Occupancy	A * B * F * G Comparative Competitors' Sales
Mar-03	100	31	66.7%	54.81	113,331	71.63	57.1%	126,792
Apr-03	100	30	95.6%	53.59	153,696	74.21	54.8%	122,001
May-03	100	31	95.8%	53.37	158,498	74.57	61.3%	141,705
Jun-03	100	30	96.2%	53.83	155,353	77.66	76.9%	179,162
Jul-03	100	31	94.5%	56.83	166,483	83.70	78.4%	203,424
Aug-03	100	31	89.2%	58.50	161,764	81.32	81.4%	205,203
Sep-03	100	30	85.2%	59.19	151,290	70.03	63.2%	132,777
Oct-03	100	31	73.7%	58.62	133,929	72.73	67.1%	151,286
Nov-03	100	30	66.0%	55.74	110,365	72.05	56.9%	122,989
Dec-03	100	31	71.8%	54.61	121,551	67.45	48.6%	101,620
Jan-04	100	31	62.9%	54.90	107,050	68.57	50.7%	107,771
Feb-04	100	29	68.8%	55.44	110,614	71.46	56.2%	116,466
Mar-04	100	31	78.8%	55.88	136,504	70.57	61.7%	134,979
Apr-04	100	30	77.4%	56.13	130,334	73.50	62.8%	138,474
May-04	100	31	82.8%	56.13	144,074	77.07	58.9%	140,722
Jun-04	100	30	84.9%	59.43	151,368	83.52	72.9%	182,658
Jul-04	100	31	73.8%	55.18	126,241	83.36	75.3%	194,587
Aug-04	100	31	82.4%	57.66	147,287	79.78	83.9%	207,500
Sep-04	100	30	77.1%	53.52	123,792	77.86	74.2%	173,316
Oct-04	100	31	80.4%	53.16	132,496	78.62	65.6%	159,882
Nov-04	100	30	64.7%	57.05	110,734	78.31	53.5%	125,688
Dec-04	100	31	45.2%	59.69	83,638	77.71	42.9%	103,347
Jan-05	100	31	39.2%	60.01	72,924	78.23	48.2%	116,891
Feb-05	100	28	63.7%	58.34	104,055	77.95	54.6%	119,170
Mar-05	100	31	74.3%	60.15	138,543	78.05	61.3%	148,318



Exhibit D

Actual Motel Sales  
March 2003 - June 2004



# Exhibit E

Comparative Occupancy Rates  
March 2003 - June 2004

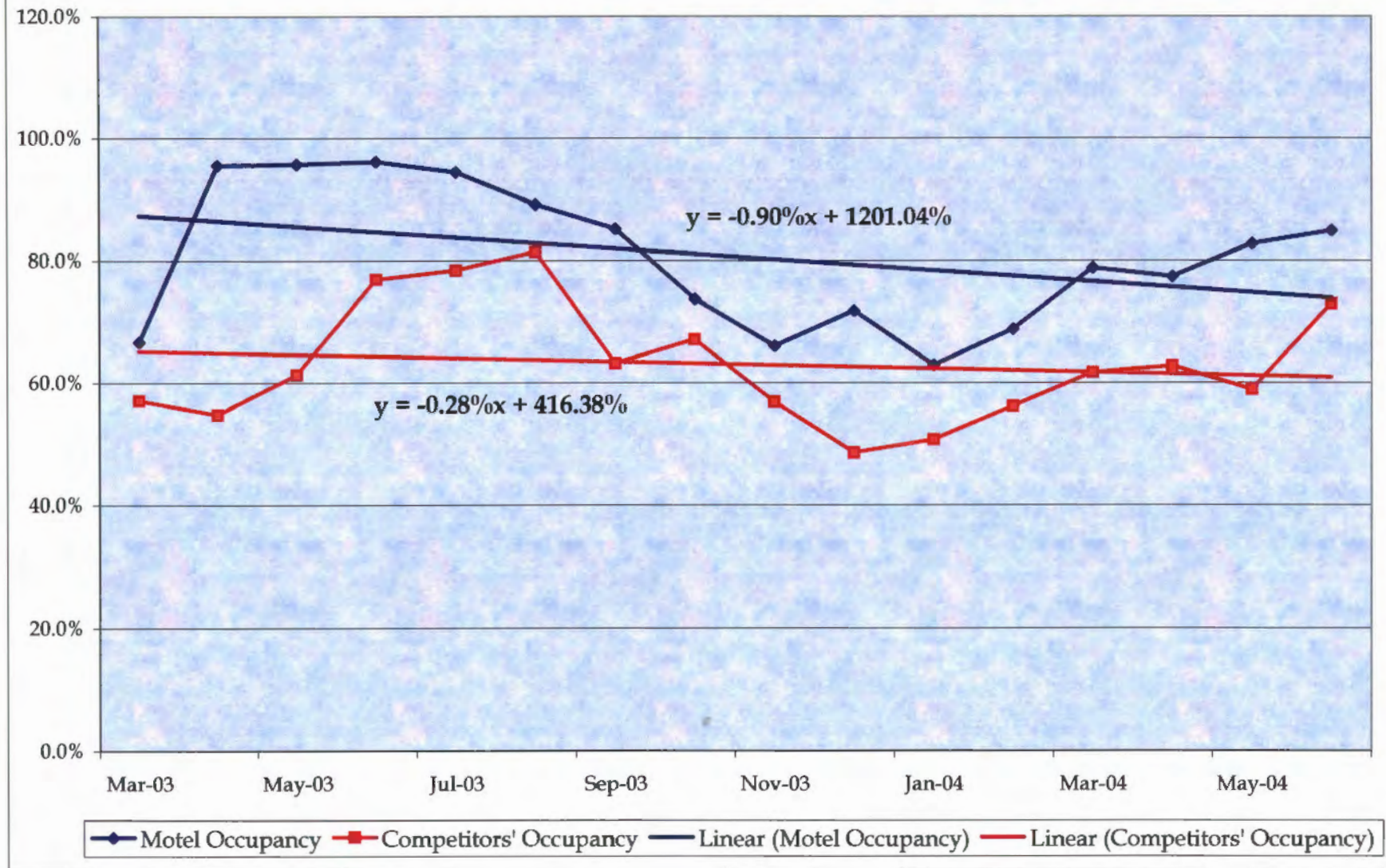




Exhibit F

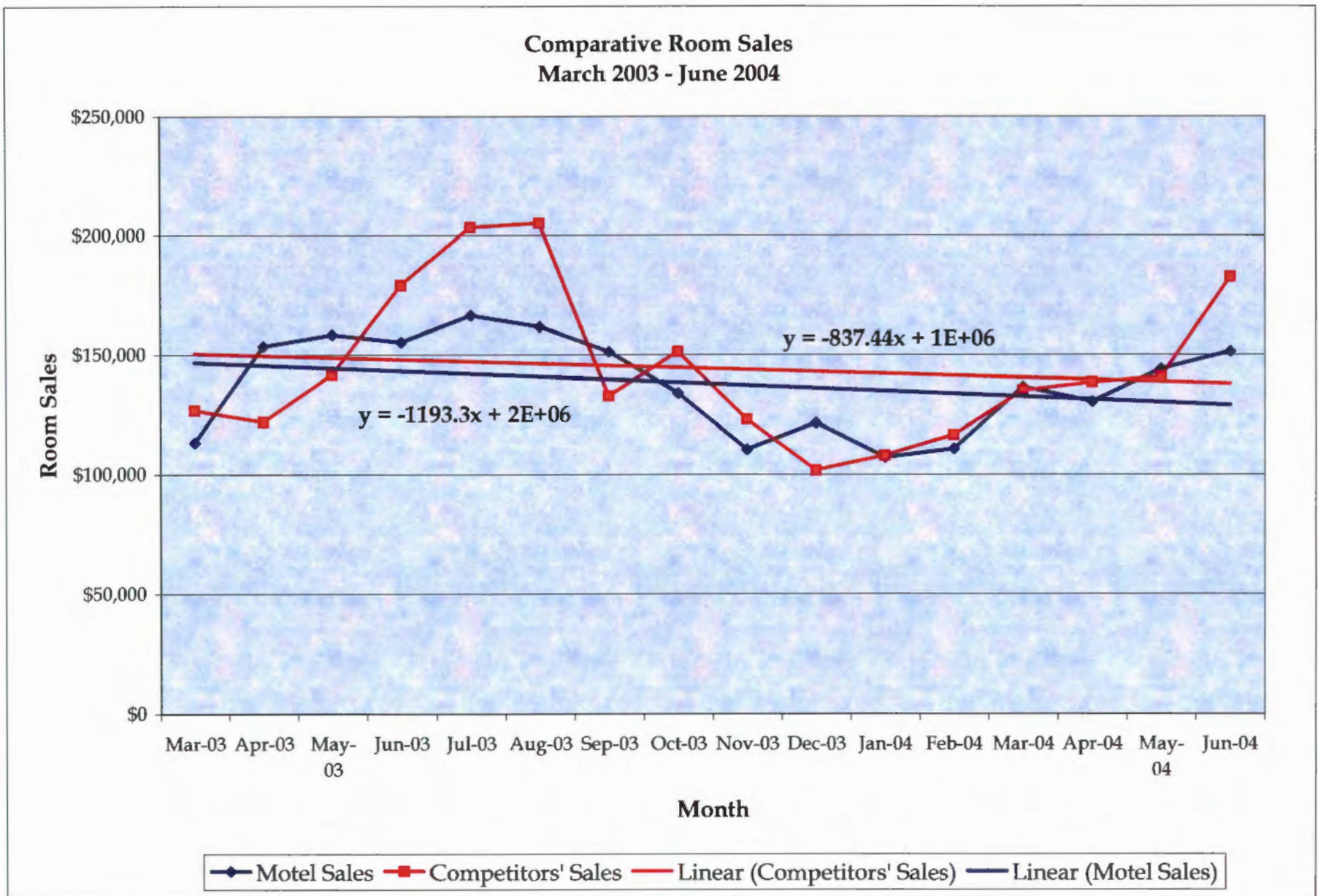


Exhibit G

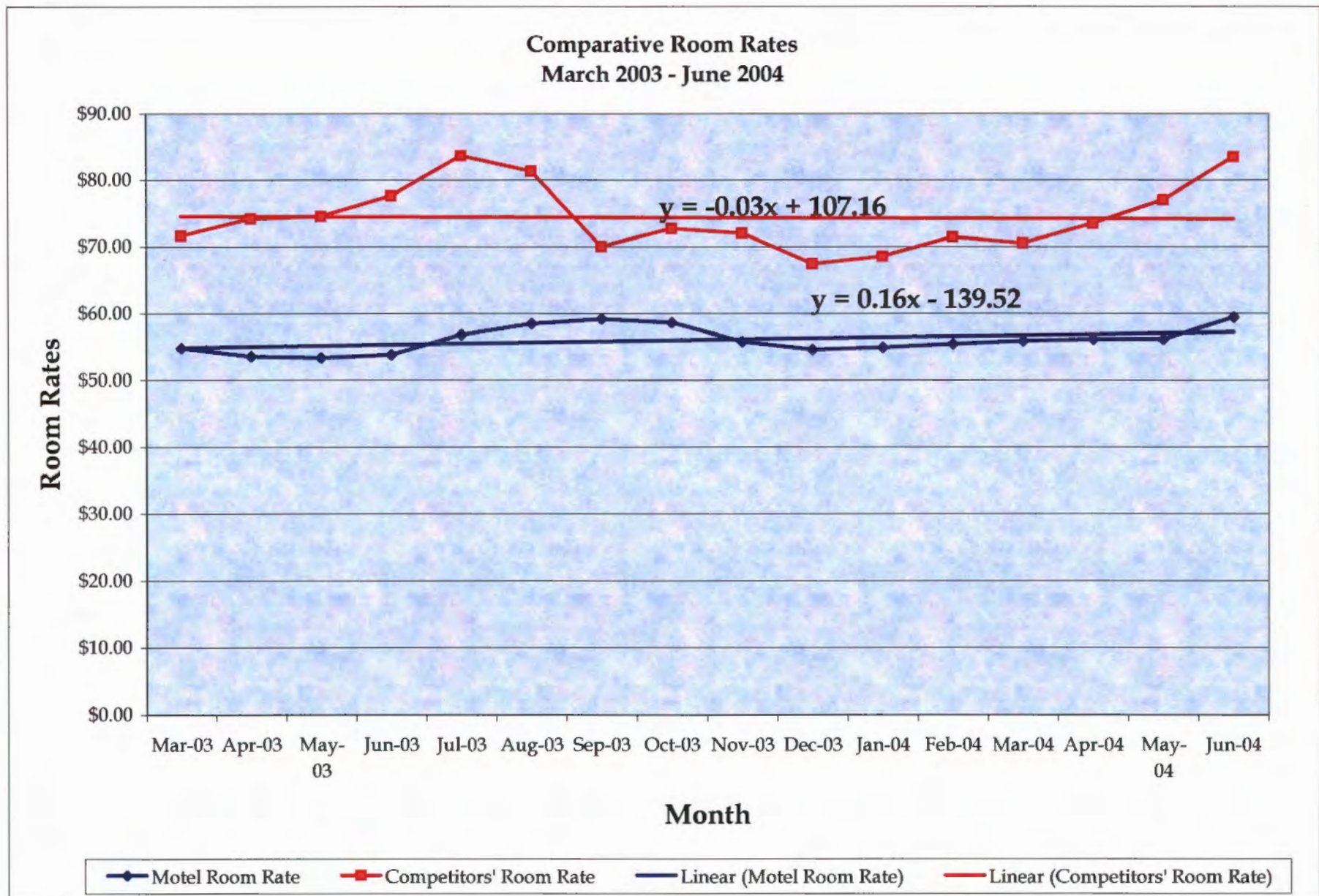




Exhibit H

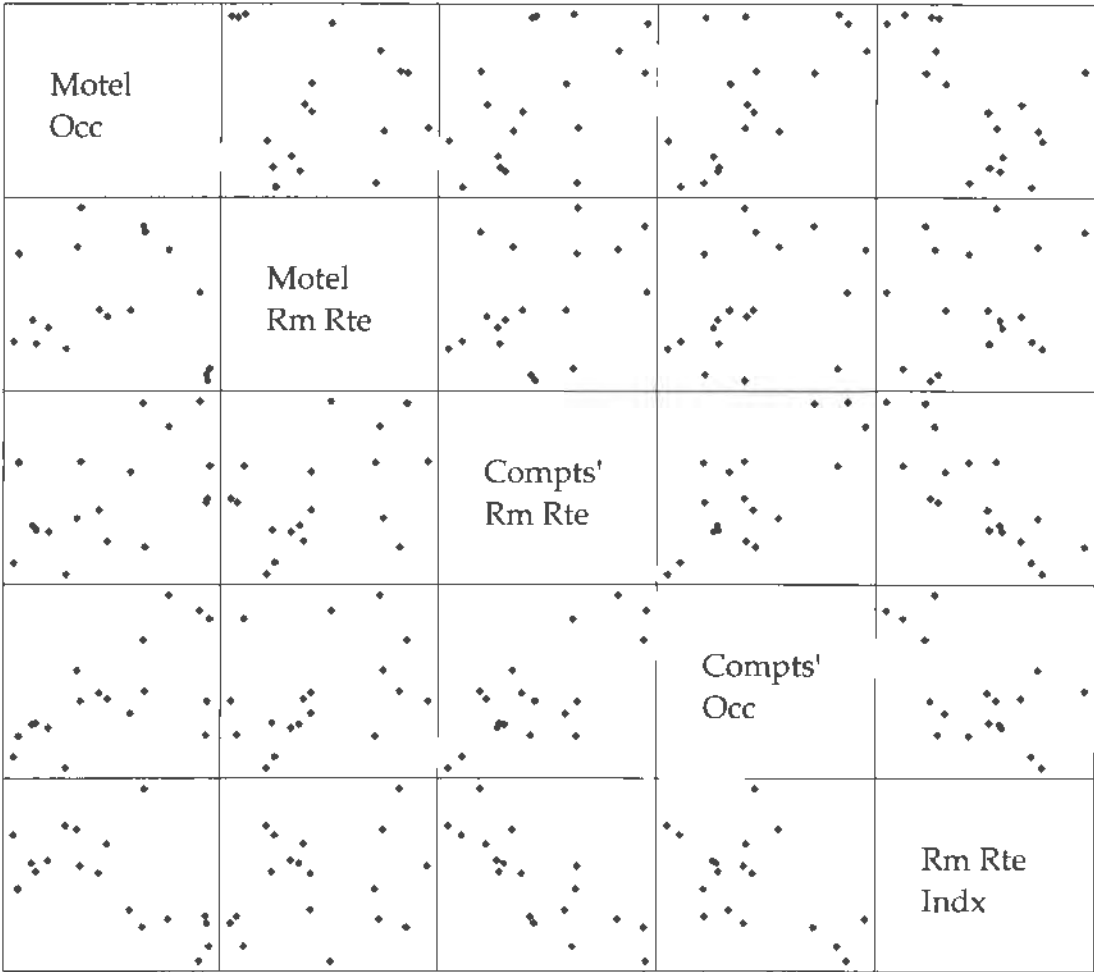


Exhibit I

## Regression Set-Up and Output

Month	Motel Occupancy	Room Rate Index	Dummy Variable	Time	Seasonal Index
Mar-03	66.70%	0.765	0	1	0.911
Apr-03	95.60%	0.722	0	2	1.043
May-03	95.80%	0.716	0	3	1.114
Jun-03	96.20%	0.693	0	4	1.132
Jul-03	94.50%	0.679	0	5	1.209
Aug-03	89.20%	0.719	0	6	1.179
Sep-03	85.20%	0.845	0	7	1.106
Oct-03	73.70%	0.806	0	8	0.982
Nov-03	66.00%	0.774	0	9	0.812
Dec-03	71.80%	0.810	0	10	0.897
Jan-04	62.90%	0.801	0	11	0.793
Feb-04	68.80%	0.776	0	12	0.822
Mar-04	78.80%	0.792	0	13	0.911
Apr-04	77.40%	0.764	0	14	1.043
May-04	82.80%	0.728	0	15	1.114
Jun-04	84.90%	0.712	0	16	1.132
Jul-04	73.80%	0.662	1	17	1.209
Aug-04	82.40%	0.723	1	18	1.179
Sep-04	77.10%	0.687	1	19	1.106
Oct-04	80.40%	0.676	1	20	0.982
Nov-04	64.70%	0.729	1	21	0.812
Dec-04	45.20%	0.768	1	22	0.897
Jan-05	39.20%	0.767	1	23	0.793
Feb-05	63.70%	0.748	0	24	0.822
Mar-05	74.30%	0.771	0	25	0.911

## SUMMARY OUTPUT

*Regression Statistics*

Multiple R	0.892
R Square	0.796
Adjusted R Square	0.755
Standard Error	0.071
COV	9.44%
Observations	25

## ANOVA

	df	SS	MS	F	Significance F
Regression	4	0.398	0.100	19.525	1.11149E-06
Residual	20	0.102	0.005		
Total	24	0.500			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.8956	0.423	2.116	0.047	0.013	1.779
Room Rate Index	-0.8164	0.427	-1.912	0.070	-1.707	0.074
Dummy Variable	-0.1352	0.045	-3.038	0.006	-0.228	-0.042
Time	-0.0032	0.003	-1.160	0.260	-0.009	0.003
Seasonal Index	0.5501	0.136	4.031	0.001	0.265	0.835

## Exhibit J

## Schedule of Forecasted and Projected Monthly Room Revenue

Year	Month	Time Period	Actual Sales	Linear Trend	Actual as a % of Trend	Seasonal Forecast	Seasonal Projection
Mar-03	3	1	113,331	139,496	81.24%	127,096	
Apr-03	4	2	153,696	139,047	110.54%	145,000	
May-03	5	3	158,498	138,598	114.36%	154,371	
Jun-03	6	4	155,353	138,148	112.45%	156,390	
Jul-03	7	5	166,483	137,699	120.90%	166,483	
Aug-03	8	6	161,764	137,250	117.86%	161,764	
Sep-03	9	7	151,290	136,801	110.59%	151,290	
Oct-03	10	8	133,929	136,352	98.22%	133,929	
Nov-03	11	9	110,365	135,903	81.21%	110,365	
Dec-03	12	10	121,551	135,454	89.74%	121,551	
Jan-04	1	11	107,050	135,005	79.29%	107,049	
Feb-04	2	12	110,614	134,556	82.21%	110,614	
Mar-04	3	13	136,504	134,107	101.79%	122,186	
Apr-04	4	14	130,334	133,658	97.51%	139,380	
May-04	5	15	144,074	133,209	108.16%	148,369	
Jun-04	6	16	151,368	132,759	114.02%	150,289	150,289
Jul-04	7	17		132,310			159,968
Aug-04	8	18		131,861			155,413
Sep-04	9	19		131,412			145,330
Oct-04	10	20		130,963			128,636
Nov-04	11	21		130,514			105,989
Dec-04	12	22		130,065			116,715
Jan-05	1	23		129,616			102,776
Feb-05	2	24	104,055	129,167	80.56%		106,184
Mar-05	3	25	138,543	128,718	107.63%		117,276

Projected Revenue - July 2004 -- January 2005

914,827

Actual Revenue - July 2004 -- January 2005

(797,111)

Lost Revenue

117,716

Month	Unadjusted Seasonal Index	Normalizing Factor	Adjusted Seasonal Index	Optimized Index	Optimizing Coefficients	
1	79.29%	0.996178	78.99%	79.29%	Intercept	139,945
2	81.38%	0.996178	81.07%	82.21%	Slope	(449)
3	96.89%	0.996178	96.52%	91.11%	RMSE =	6,490
4	104.02%	0.996178	103.63%	104.28%	RSQ =	0.904
5	111.26%	0.996178	110.83%	111.38%		
6	113.24%	0.996178	112.80%	113.20%	Actual Sales - July '03 - Jan '04	952,432
7	120.90%	0.996178	120.44%	120.90%	Forecast - July '04 - Jan '05	914,827
8	117.86%	0.996178	117.41%	117.86%	Delta - \$	37,605
9	110.59%	0.996178	110.17%	110.59%	Delta - %	3.9%
10	98.22%	0.996178	97.85%	98.22%		
11	81.21%	0.996178	80.90%	81.21%		
12	89.74%	0.996178	89.39%	89.74%		
12			100.0%	100.0%		

Exhibit K

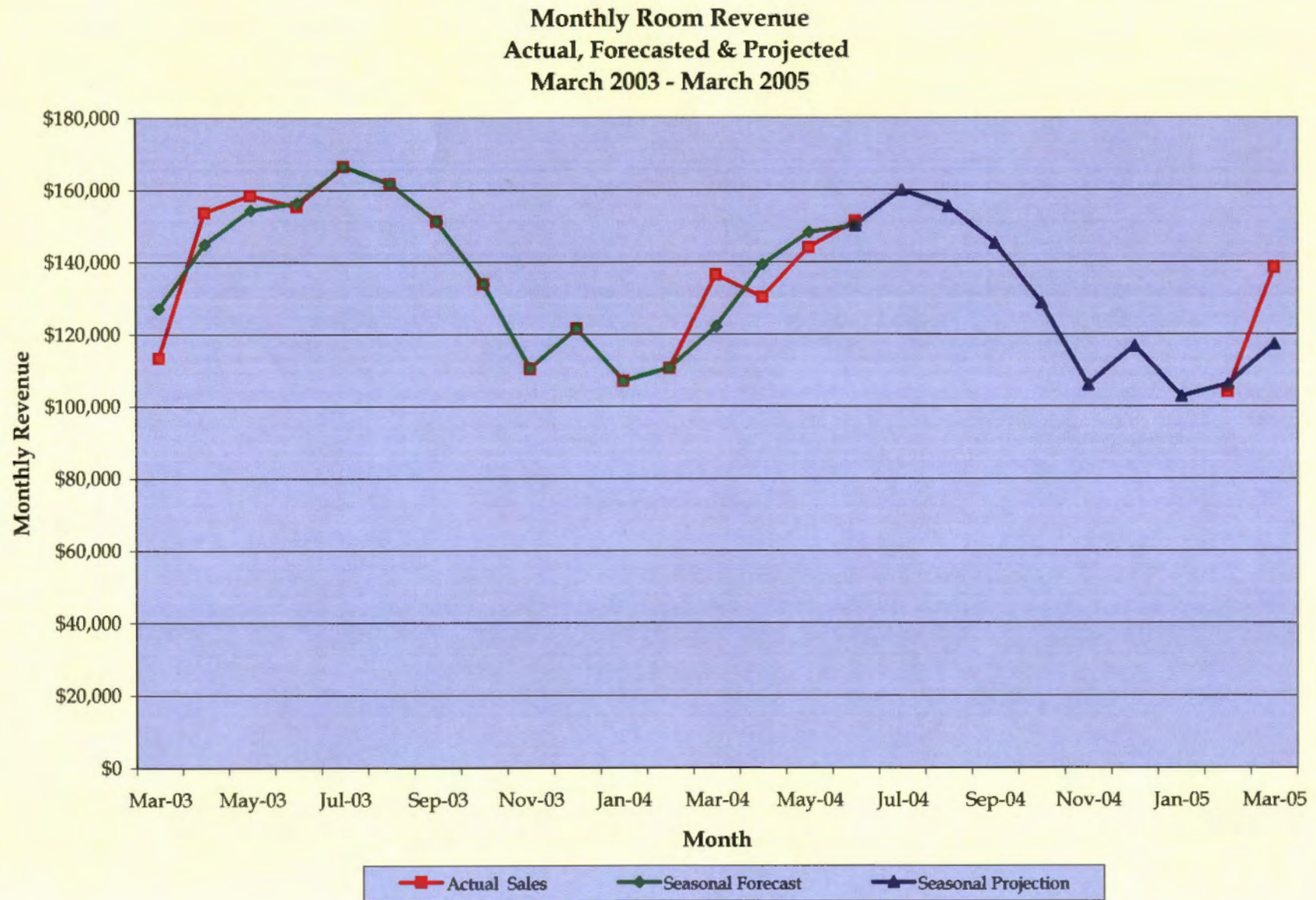




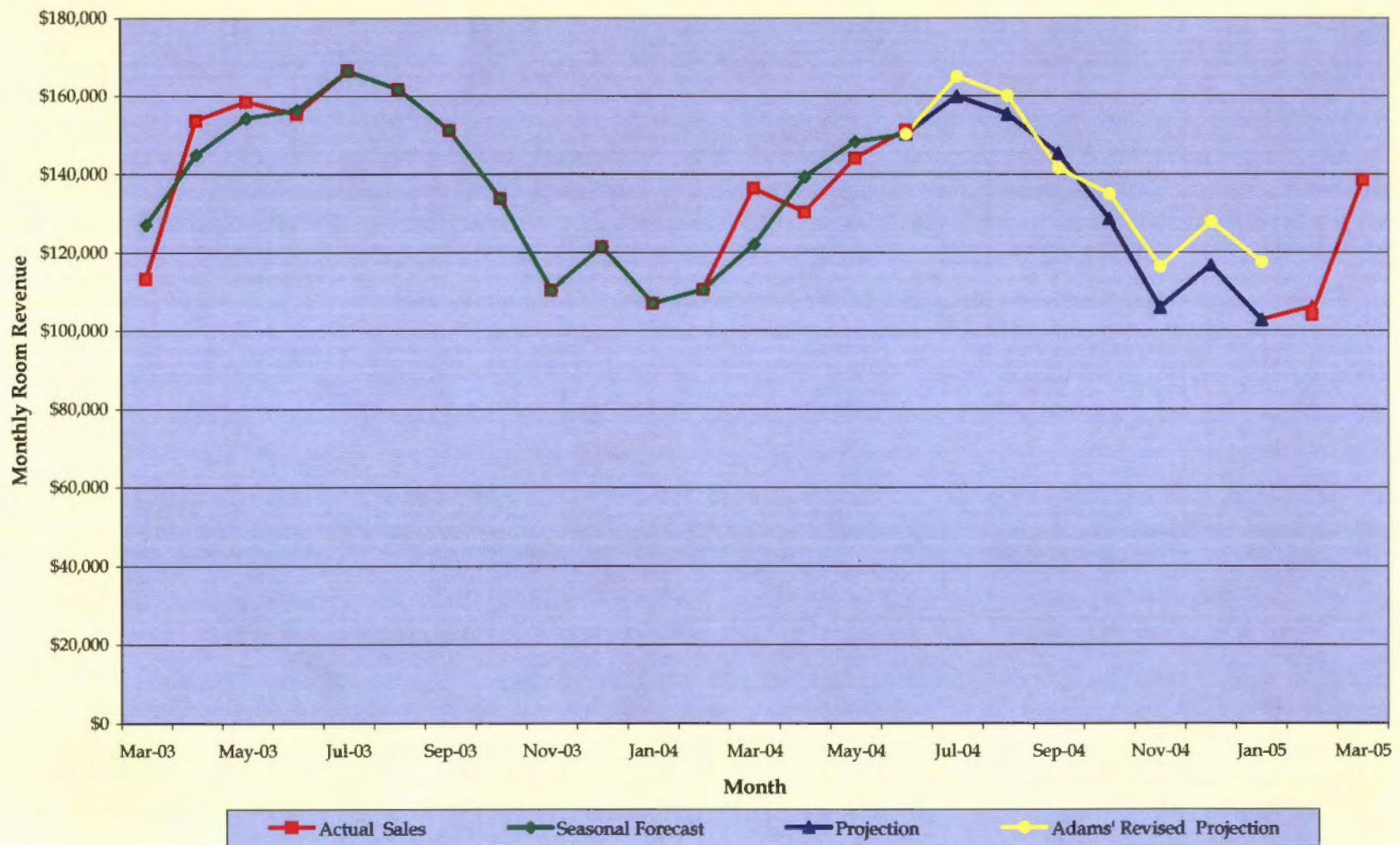
Exhibit L

**Comparative Monthly Room Revenue**  
**Actual, Forecasted, Projected & Adams' Projection**  
**March 2003 - March 2005**



Exhibit M

**Comparative Monthly Room Revenue**  
**Actual, Forecasted, Projected & Adams' Projection**  
**March 2003 - March 2005**





# Exhibit N

## Paired Samples Using *t*

### Input:

<i>Months in the POI</i>	<i>Filler</i>	<i>Adams-Revised</i>
July-04	159,968	165,063
August-04	155,413	160,077
September-04	145,330	141,500
October-04	128,636	135,029
November-04	105,989	116,404
December-04	116,715	127,960
January-05	102,776	117,546
Total	914,827	963,579

Null hypothesis: There is no difference between the forecast means.

Alternate hypothesis: There is a difference between the forecast means.

### Output: t-Test: Paired Two Sample for Means

	<i>Filler</i>	<i>Adams-Revised</i>
Mean	130,690	137,654
Variance	545,262,355	370,933,825
Observations	7	7
Pearson Correlation	0.978	
Hypothesized Mean Difference	0	
Confidence Level	0.95	
df	6	
t Stat	-3.066	
P(T<=t) one-tail	0.011	
t Critical one-tail	1.943	
P(T<=t) two-tail	0.022	
t Critical two-tail	2.447	